1.6 A LIBRARY OF PARENT FUNCTIONS

What You Should Learn

- Identify and graph linear and squaring functions.
- Identify and graph cubic, square root, and reciprocal function.
- Identify and graph step and other piecewise-defined functions.
- Recognize graphs of parent functions.

Linear and Squaring Functions

Linear and Squaring Functions

The graph of the **linear function** $f(x) = ax + b$ is a line with slope $m = a$ and y-intercept at $(0, b)$.

Linear and Squaring Functions

Characteristics of the Linear Function:

- Domain: the set of all real numbers.
- Range: the set of all real numbers.
- x-intercept: $(-b/m, 0)$
- y-intercept: $(0, b)$.
- The graph is increasing if $m > 0$, decreasing if $m < 0$, and constant if $m = 0$.

Example 1 – *Writing a Linear Function*

Write the linear function for which $f(1) = 3$ and $f(4) = 0$.

Solution:

To find the equation of the line that passes through $(x_1, y_1) = (1, 3)$ and $(x_2, y_2) = (4, 0)$ first find the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{4 - 1}$$

$$= \frac{-3}{3}$$

$$= -1$$

Example 1 – *Solution*

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Next, use the point-slope form of the equation of a line.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 3 = -1(x - 1)$ Substitute for x_1, y_1 and m

$y = -x + 4$ Simplify.

$f(x) = -x + 4$ Function notation

Example 1 – *Solution*

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The graph of this function is shown in Figure 1.65.

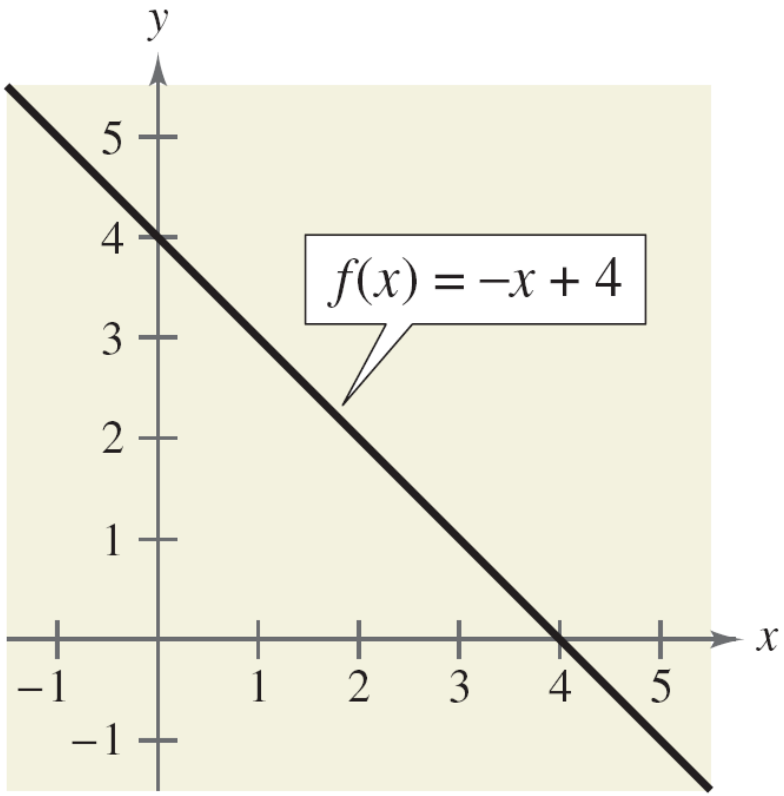


Figure 1.65

Linear and Squaring Functions

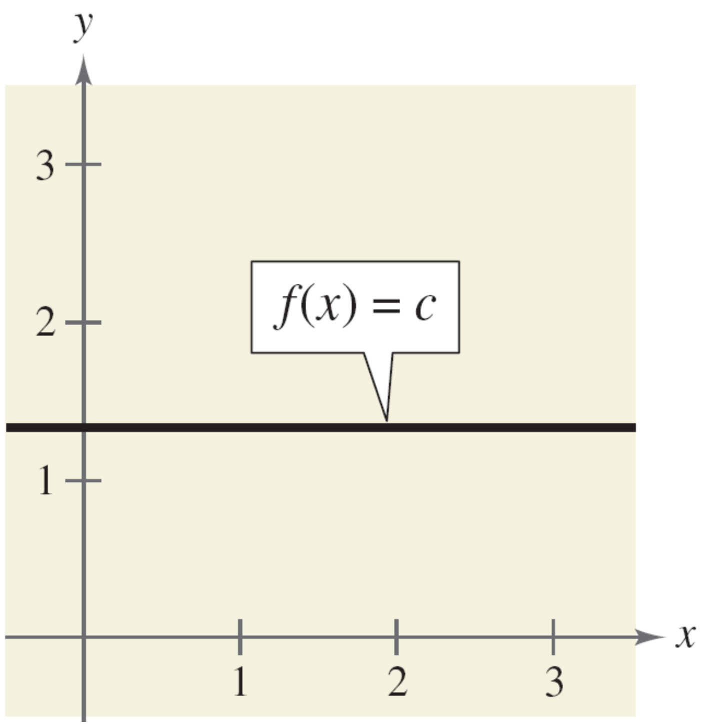
Constant Function

$$f(x) = c$$

Domain : all real numbers

Range: a single real number *c*.

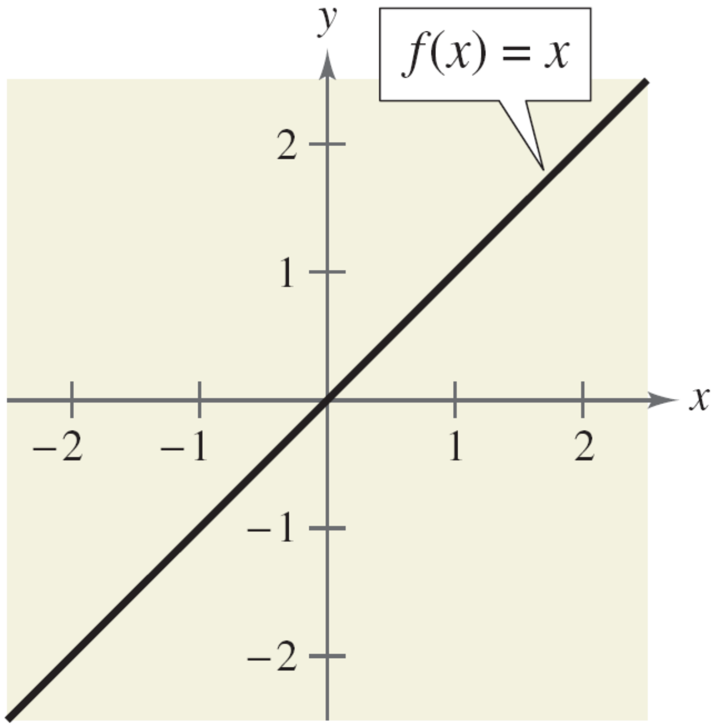
Graph: a horizontal line



Linear and Squaring Functions

Identity Function

$$f(x) = x$$



Domain: the set of all real numbers

Range: the set of all real numbers

Graph: a line with $m = 1$ and a y-intercept at (0, 0).

a line for which each x-coordinate equals the corresponding y- coordinate.

increasing

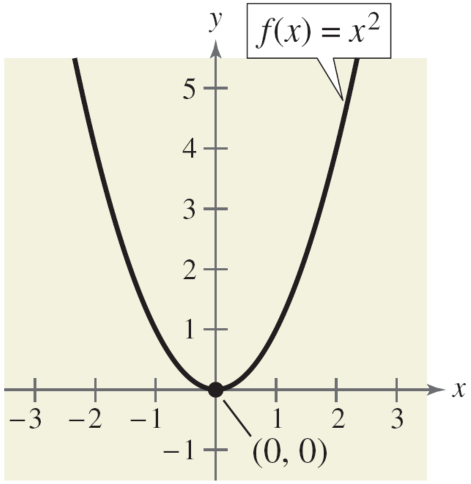
Linear and Squaring Functions

Squaring Function

$$f(x) = x^2$$

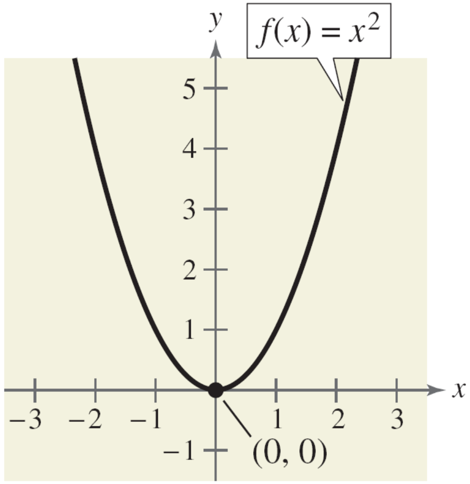
Graph: U-shaped curve with the following characteristics.

- The domain of the function is the set of all real numbers.
- The range of the function is the set of all nonnegative real numbers.
- The function is even.



Linear and Squaring Functions

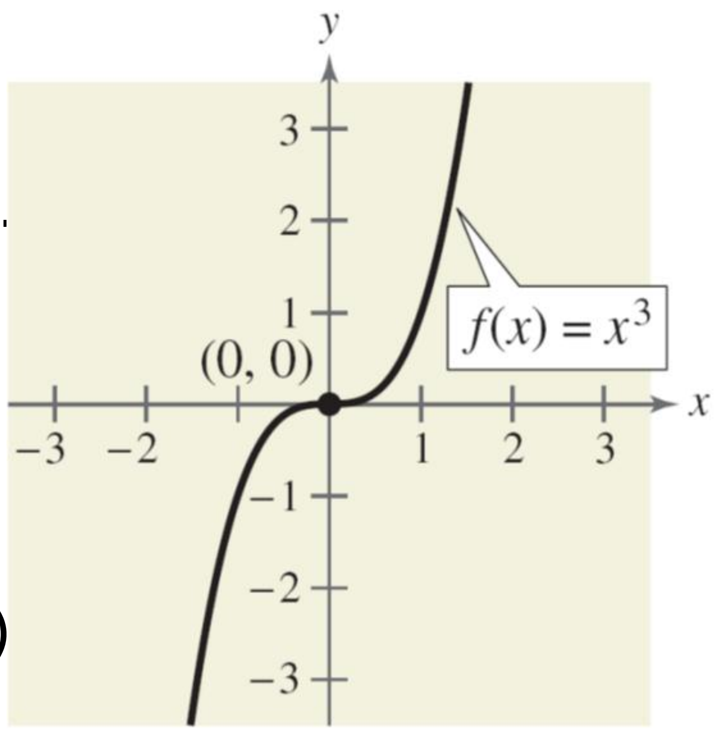
- The graph has an intercept at (0, 0).
- The graph is decreasing on the interval $(-\infty, 0)$ and increasing on the interval $(0, \infty)$
- The graph is symmetric with respect to the *y*-axis.
- The graph has a relative minimum at (0, 0).



Cubic, Square Root, and Reciprocal Functions

Cubic, Square Root, and Reciprocal Functions

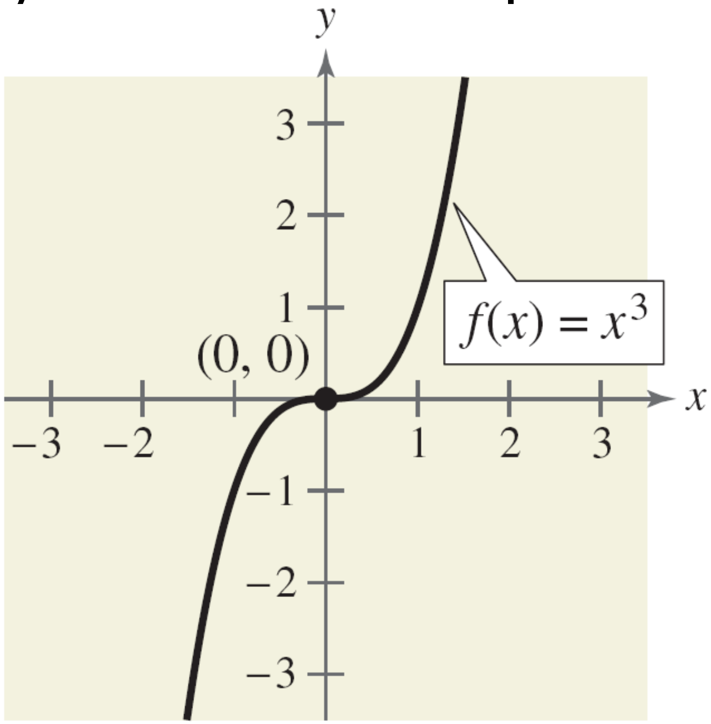
1. The graph of the ***cubic* function** $f(x) = x^3$ has the following characteristics.



- Domain: the set of all real numbers.
- Range: the set of all real numbers.
- The function is odd.
- The graph has an intercept at (0, 0).

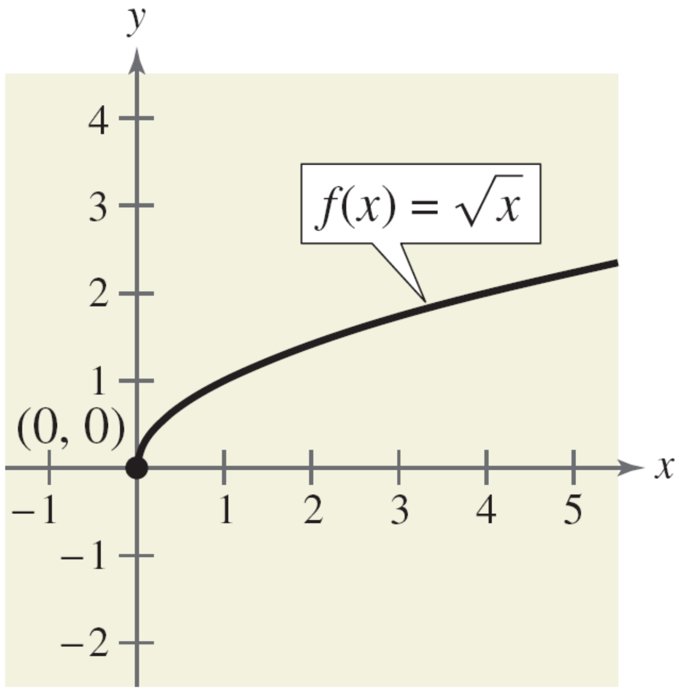
Cubic, Square Root, and Reciprocal Functions

- The graph is increasing on the interval $(-\infty, \infty)$.
- The graph is symmetric with respect to the origin.

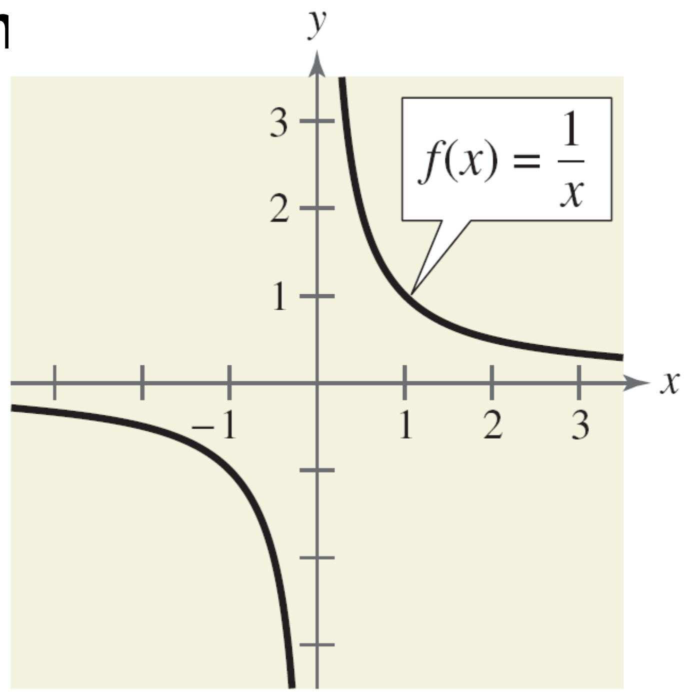


2. The graph of the ***square root*** function $f(x) = \sqrt{x}$ has the following characteristics.

- Dmain: the set of all nonnegative real numbers.
- Range: the set of all nonnegative real numbers.
- The graph has an intercept at (0, 0).
- The graph is increasing on the interval $(0, \infty)$



3. The graph of the ***reciprocal* function** following characteristics.



- Domain: $(-\infty, 0) \cup (0, \infty)$
- Range: $(-\infty, 0) \cup (0, \infty)$
- Odd.
- No intercepts.
- The graph is decreasing on the intervals $(-\infty, 0)$ and $(0, \infty)$.
- The graph is symmetric with respect to the origin.

Step and Piecewise-Defined Functions

Step and Piecewise-Defined Functions

Functions whose graphs resemble sets of stairsteps are known as **step functions.**

greatest integer function,

$f(x) = [\![x]\!]$ = *the greatest integer less than or equal to x*.

Step and Piecewise-Defined Functions

$[\![-1]\!]$ = (greatest integer ≤ –1) = –1

$[\![-\frac{1}{2}]\!]$ = (greatest integer ≤ $-\frac{1}{2}$) = –1

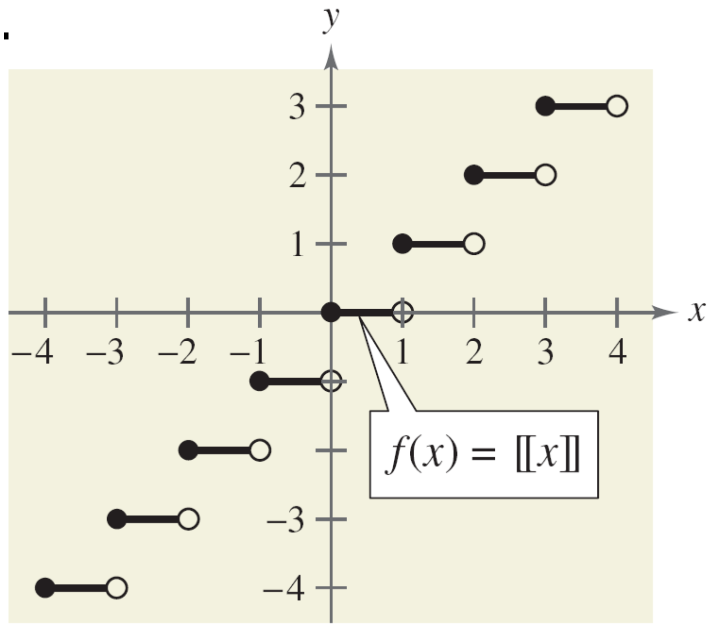
$[\![\frac{1}{10}]\!]$ = (greatest integer ≤ $\frac{1}{10}$) = 0

$[\![1.5]\!]$ = (greatest integer ≤ 1.5) = 1

Step and Piecewise-Defined Functions

$f(x) = [\![x]\!]$

- Domain: the set of all real numbers.
- Range of: the set of all integers.
- *y*-intercept: (0, 0)
- *x*-intercepts: interval [0, 1).



Example 2 – *Evaluating a Step Function*

Evaluate the function when $x = -1$, 2 and $\frac{3}{2}$.

$$f(x) = [\![x]\!] + 1$$

Solution:

For $x = -1$, the greatest integer ≤ -1 is -1, so

$$f(-1) = [\![-1]\!] + 1$$

$$= -1 + 1$$

$$= 0$$

Example 2 – *Solution*

cont'd

For x = 2, the greatest integer ≤ 2 is 2, so

$$f(2) = [\![2]\!] + 1$$

$$= 2 + 1$$

$$= 3.$$

For $x = \frac{3}{2}$, the greatest integer $\le \frac{3}{2}$ is 1, so

$$f\left(\frac{3}{2}\right) = \left[\!\left[\frac{3}{2}\right]\!\right] + 1$$

$$= 1 + 1$$

$$= 2$$

Example 2 – *Solution*

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You can verify your answers by examining the graph of $f(x) = [\![x]\!] + 1$ shown in Figure 1.73.

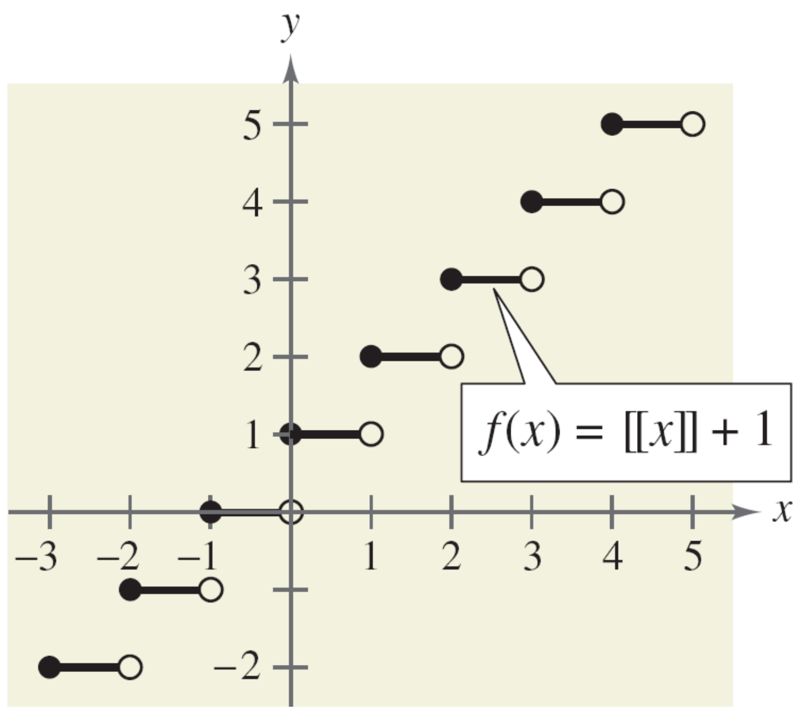
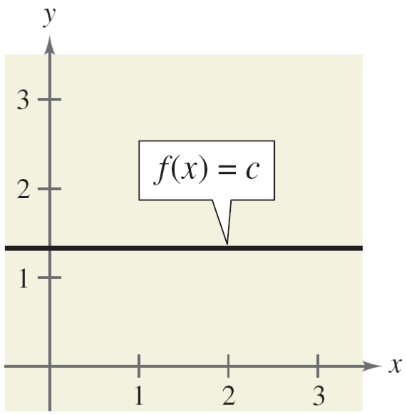


Figure 1.73

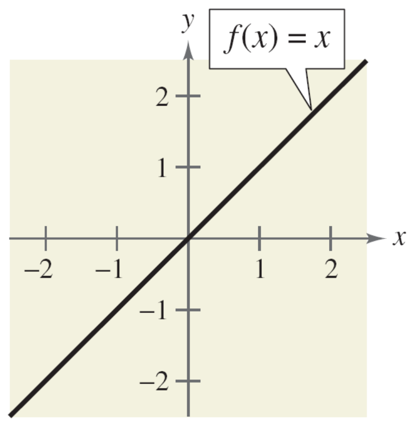
Parent Functions

Parent Functions

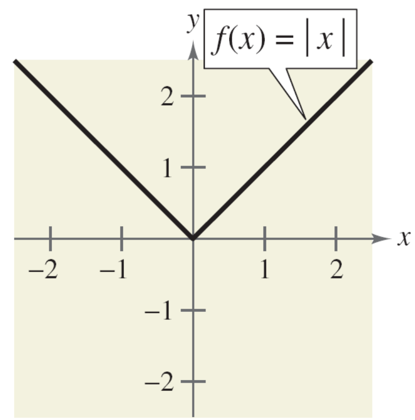
The eight graphs shown in Figure 1.75 represent the most commonly used functions in algebra.



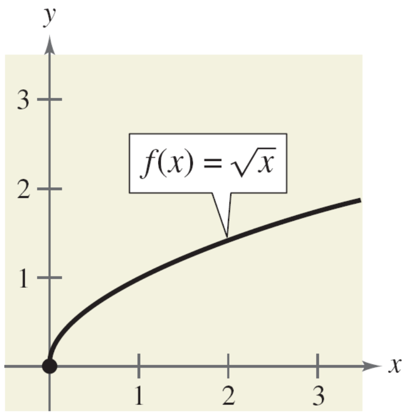
(a) Constant Function



(b) Identity Function



(c) Absolute Value Function



(d) Square Root Function

Parent Functions

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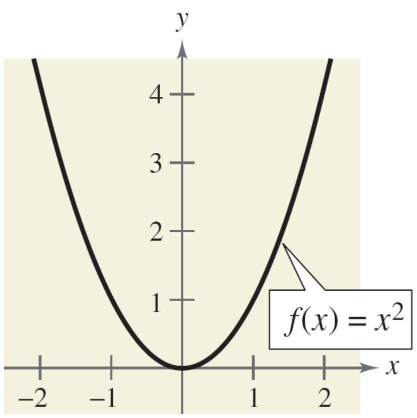
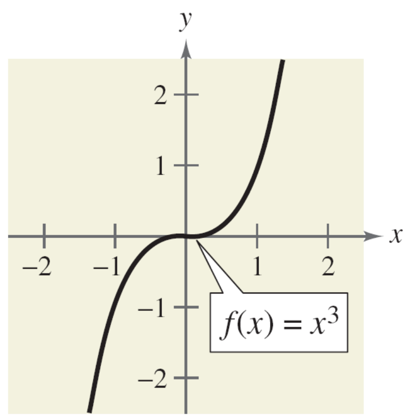
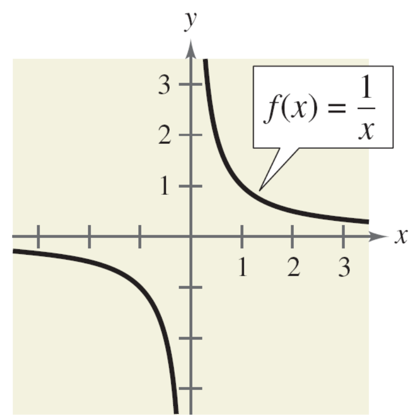


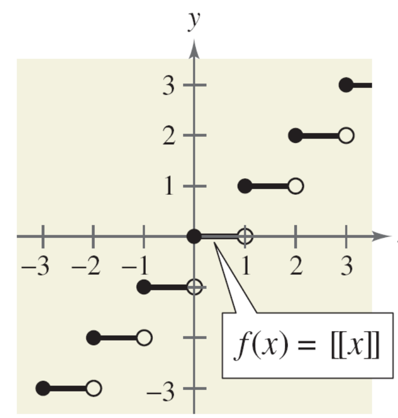
(e) Quadratic Function



(f) Cubic Function



(g) Reciprocal Function



(h) Greatest Integer Function

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