What You Should Learn

• Use vertical and horizontal shifts to sketch graphs of functions.

• Use reflections to sketch graphs of functions.

• Use nonrigid transformations to sketch graphs of functions.
Shifting Graphs
Shifting Graphs

\[ h(x) = x^2 + 2 \]

Start with the graph of \( f(x) = x^2 \)

*Shifting upward two units*

\[ h(x) = x^2 + 2 = f(x) + 2 \]

Upward shift of two units
$g(x) = (x - 2)^2$

Start with the graph of $f(x) = x^2$

*Shifting* to the *right* two units

$g(x) = (x - 2)^2 = f(x - 2)$

Right shift of two units
Vertical and Horizontal Shifts

Let $c$ be a positive real number. **Vertical and horizontal shifts** in the graph of $y = f(x)$ are represented as follows.

1. Vertical shift $c$ units **upward**: $h(x) = f(x) + c$
2. Vertical shift $c$ units **downward**: $h(x) = f(x) - c$
3. Horizontal shift $c$ units to the **right**: $h(x) = f(x - c)$
4. Horizontal shift $c$ units to the **left**: $h(x) = f(x + c)$
Example 1 – *Shifts in the Graphs of a Function*

Use the graph of \( f(x) = x^3 \) to sketch the graph of each function.

a. \( g(x) = x^3 - 1 \)  
   b. \( h(x) = (x + 2)^3 + 1 \)

**Solution:**

a. Relative to the graph of \( f(x) = x^3 \), the graph of

\[
g(x) = x^3 - 1
\]

is a downward shift of one unit, as shown in Figure 1.78.
Example 1 – Solution

b. Relative to the graph of $f(x) = x^3$, the graph of

$$h(x) = (x + 2)^3 + 1$$

involves a left shift of two units and an upward shift of one unit, as shown in figure.
Reflecting Graphs
Reflecting Graphs

\[ h(x) = -x^2 \]

is the mirror image (or reflection) of the graph of

\[ f(x) = x^2, \]
Reflecting Graphs

**Reflections in the Coordinate Axes**

**Reflections** in the coordinate axes of the graph of \( y = f(x) \) are represented as follows.

1. Reflection in the \( x \)-axis: \( h(x) = -f(x) \)
2. Reflection in the \( y \)-axis: \( h(x) = f(-x) \)
Example 2 – Finding Equations from Graphs

The graph of the function given by

\[ f(x) = x^4 \]

is
Example 2 – Finding Equations from Graphs

Each of the graphs in figure is a transformation of the graph of \( f \). Find an equation for each of these functions.

(a) \( y = g(x) \)

(b) \( y = h(x) \)
Example 2 – Solution

a. The graph of $g$ is a reflection in the $x$-axis followed by an upward shift of two units of the graph of $f(x) = x^4$.

So, the equation for $g$ is

$$g(x) = -x^4 + 2.$$  

b. The graph of $h$ is a horizontal shift of three units to the right followed by a reflection in the $x$-axis of the graph of $f(x) = x^4$.

So, the equation for $h$ is

$$h(x) = -(x - 3)^4.$$
Reflecting Graphs

Function involving \textcolor{red}{\textbf{square roots}}, restrict the domain to \textbf{EXCLUDE} negative numbers inside the radical.

\begin{align*}
\text{Domain of } g(x) &= -\sqrt{x}: \quad x \geq 0 \\
\text{Domain of } h(x) &= \sqrt{-x}: \quad x \leq 0 \\
\text{Domain of } k(x) &= -\sqrt{x + 2}: \quad x \geq -2
\end{align*}
Nonrigid Transformations
Nonrigid Transformations

**Rigid** transformations: the basic shape of the graph is unchanged.

- Horizontal shifts,
- Vertical shifts,
- Reflections

Change only the *position* of the graph in the coordinate plane.

**Nonrigid transformations**: cause a *distortion*—a change in the shape of the original graph.
A nonrigid transformation of the graph of $y = f(x)$ is represented by

$$g(x) = cf(x),$$

where the transformation is a **vertical stretch** if $c > 1$ and a **vertical shrink** if $0 < c < 1$.

A nonrigid transformation of the graph of $y = f(x)$ is represented by

$$h(x) = f(cx),$$

where the transformation is a **horizontal shrink** if $c > 1$ and a **horizontal stretch** if $0 < c < 1$. 
Example 4 – Nonrigid Transformations

Compare the graph of each function with the graph of \( f(x) = |x| \).

**a.** \( h(x) = 3|x| \)  
**b.** \( g(x) = \frac{1}{3}|x| \)

**Solution:**

**a.** Relative to the graph of \( f(x) = |x| \), the graph of \( h(x) = 3|x| = 3f(x) \) is a vertical stretch (each \( y \)-value is multiplied by 3) of the graph of \( f \).
Example 4 – Solution

b. Similarly, the graph of

\[ g(x) = \frac{1}{3}|x| = \frac{1}{3}f(x) \]

is a vertical shrink (each \(y\)-value is multiplied by \(\frac{1}{3}\)) of the graph of \(f\).