OPTIMAL ALLOCATION OF MULTIPLE EMERGENCY SERVICE RESOURCES FOR CRITICAL TRANSPORTATION INFRASTRUCTURE PROTECTION

Yongxi Huang
Email: yxhuang@ucdavis.edu
Department of Civil and Environmental Engineering
Institute of Transportation Studies
University of California
Davis, CA 95616

Yueyue Fan (Corresponding Author)
Phone: 530-754-6408
Fax: 530-752-7872
Email: yyfan@ucdavis.edu
Department of Civil and Environmental Engineering
Institute of Transportation Studies
University of California
Davis, CA 95616

Ruey Long Cheu
Department of Civil and Environmental Engineering
The University of Texas at El Paso
El Paso, Texas 79912
United States

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Abstract: Optimal deployment of limited emergency resources in a large-scale area is of interests to public agencies at all levels. In this paper, the problem of allocating limited emergency service vehicles including fire engines, fire trucks and ambulances among a set of candidate stations is formulated as a mixed integer linear programming model, in which the objective is to maximize the service coverage to the Critical Transportation Infrastructures (CTIs). Using this model, we study the effects of demand at CTI nodes and transportation network performance on the optimal coverage to CTIs. In addition, given a fixed total budget, we are also interested in identifying the most efficient distribution of investment among the three types of emergency service vehicles. In order to cope with the uncertainty involved in some of the model parameters such as the traffic network performance, formulations based on different risk preferences are proposed. The concept of “regret” is applied to evaluate the robustness of proposed resource allocation strategies. The applicability of the proposed methodologies to high-density metropolitan area is demonstrated through a case study using data from current practice in Singapore.

Keywords: critical transportation infrastructure protection, optimal resource allocation, facility location, mixed-integer programming
1. INTRODUCTION

Critical Transportation Infrastructures (CTIs), such as bus terminals and interchanges, mass rapid transit (MRT) stations, tunnels, airports and seaports, are vital for maintaining normal societal functionality, especially in metropolitan areas. On the other hand, these facilities are vulnerable to natural and human-made disasters due to the density of people and traffic at these locations and their high repairing and maintenance costs. Therefore, developing an effective protection mechanism for these CTIs is important in disaster mitigation and protection of large urban areas.

One way to protect the CTIs is to improve emergency response readiness. This requires sufficient amount of emergency service resources to be able to serve CTIs within acceptable time. In this paper, we focus on optimal allocation of fire engines, fire trucks and ambulances to protect CTIs, but the methods presented here are applicable to other emergency service resources as well, e.g. locate Emergenc Medical Service (EMS) vehicles. Ideally we want to deploy as much resource as we can to protect the CTIs. However, service resources are often limited in reality. Thus the question becomes how to allocate limited resources to a set of possible stations in order to serve as many CTIs as possible, i.e., to maximize the service coverage to the CTIs. A CTI node is considered to be covered only if all three types of vehicles can simultaneously reach the node within required time window (service standard) and is available at a specified reliability level (service reliability). Detailed definitions of service standard and reliability are given in Section 2.

The problem being addressed in this paper belongs to the general category of facility location problem, whose formulations and solution algorithms have been studied extensively in operations research over last several decades. A thorough review on strategic facility location problems is provided by Schilling et al. [1] and Owen and Daskin [2]. Static and deterministic facility location problems can conventionally be further classified into three basic types in terms of their different objectives as well as constraints and summarized in Table 1 [3]. For a comprehensive review and implementations of the three types of problems in emergency service location problem, readers are referred to [4].

The covering model (the first type of model in Table 1) is adopted in this study to locate emergency service facilities (e.g. fire engines, fire trucks and ambulances). The reasons lie in twofold: in reality, acceptable service standards in terms of travel time for fire engines, fire trucks and ambulances are usually predetermined by emergency management agencies and naturally become the constraints in our model; this study is targeted to maximize coverage to CTI nodes (i.e. a CTI node is said to be covered if it can be served within a specified time) with limited emergency resources. Earlier models as summarized in Table 1 do not consider the possibility of a server being unavailable when it is busy serving other demand. Later, additional constraints are imposed to guarantee that the probability of at least one vehicle being available to serve each demand node must be greater than or equal to a predefined constant $\alpha$. Daskin [22] proposed the concept of system-wide busy fraction in his model to maximize the expected coverage within a time standard, given $p$ number of facilities (stations) to be located in the network. Bianchi and Church [23] proposed a hybrid model which incorporates the concepts of MEXCLP [22] and FLEET [6] to site stations and allocate ambulances. This model has been applied to locate EMS vehicles in Fayetteville, North Carolina [24]. ReVelle and Marianov [25] then applied the concept of busy fraction of servers to problem of allocating multiple types of emergency resources. ReVelle and Hogan [21, 26] examined a similar problem from different
aspects, where the objective is to minimize the total number of utilized servers subject to server availability constraints.

Obviously, allocating emergency service resource is a planning problem, which heavily involves prediction of model parameters such as incident rate at demand sites and the transportation network performance. In reality, these parameters are often random. How to treat uncertainty in location problem has become a topical subject recently. A comprehensive review on facility location problems under uncertainty is provided by Snyder [27]. In the field of stochastic system optimization, various decision criteria have been introduced to cope with uncertainty, including expectation, reliability, robustness, and use of chance constraints and penalty functions. Most commonly used criterion is the average performance (expectation). In general, expectation based strategies are risk neutral and tend to perform well in the long run in a repetitive environment. However, some times random events do not repeat themselves often. Large scale infrastructure planning against future disasters is a good example of such one-time decision making. Robust approaches are hence introduced to handle decision making under extreme uncertain environments. The concept of “regret” is used in robust optimization to measure the difference between the best possible result if every thing could be predicted in advance and the actually realized result [28]. Since robust optimization focuses on the worst case scenario, it is usually more conservative than techniques focusing on expectation. Location problems studied in robust optimization framework can be found in references [4, 20]. In general, the choice of decision criteria usually reflects and depends on decision makers’ risk preferences. In the later part of this paper, we will analyze the performance of different risk models in various uncertain environments, and will explore the impact of different risk preferences on the effectiveness of emergency service resource allocation decisions.

This study is built on the foundation of our previous deterministic model [29]. We have revised the previous model from the binary integer linear programming formulation to a mixed integer linear program. This change speeds up model computation and facilitates extensive sensitivity analysis of the model. More importantly, we provide a thorough sensitivity and robustness analysis of the model in order to explore the applicability of the model in practice especially in large-scale metropolitan areas. Sensitivity analysis allows us to find how changes in some predefined model parameters, such as the amount of emergency service resources, will affect the maximum coverage. This type of cost-benefit analysis is critical to decision makers since all requests for federal or state funding need to be justified. Robust analysis emphasizes more on parameters describing the uncertain environment, for example, the roadway travel time and demand frequency. The key goal of robust analysis is to provide a broader decision support under environments of different uncertainty levels and to inform decision makers of the effect of different risk preferences.

In the following sections, we will first give the expectation-based formulation for the emergency service resource allocation problem, and then present a robust optimization formulation for the same problem. Lastly, we will interpret the model results and draw policy implications of the model via a case study based on data from Singapore.

2. MATHEMATICAL MODELS

Given predicted demand for emergency service at CTI nodes, we want to find an optimal strategy for allocating limited number of fire engines, fire trucks and ambulances to a set of pre-defined candidate stations so as to maximize the coverage of the CTIs. A CTI node is covered
when it is served within required time by at least one fire engine, one fire truck and one ambulance. In addition, the service reliability, defined as the probability of at least one vehicle of each type being available at any time, is required to be no less than $\alpha$. This is a maximum expected covering problem. Models considering service reliability are categorized as probabilistic models in the review by Owen and Daskin [1]. However, we shall realize the difference between standard stochastic models and the proposed model. Standard stochastic models usually involve explicit probability distribution of random parameters or possible scenarios, while the proposed maximum expected covering model preprocesses input parameters based on reliability requirement and historical demand quantity and then input all parameters into the core model as known deterministic values.

2.1 Base Model Formulation

Let us first consider formulation of the base scenario, in which the following assumptions are made:

1. The total number of available emergency vehicles of each type is given.
2. A set of candidate stations is pre-defined and their locations are known.
3. Capacity restriction is imposed at each station.
4. Incident occurrence rates at demand nodes are estimated based on historic data.
5. Emergency service vehicles are assumed to travel at their free flow speeds. Note that the free flow speeds of emergency service vehicles are higher than that of the normal vehicles.

The main purpose of introducing the base model is to explain the concept and computation of service reliability.

Denote $I$ as the set of demand nodes (CTI nodes in this paper) and $J$ as the set of possible stations. Let us first consider the availability of each type of emergency service vehicle at station $j$ ($j \in J$) when there is a request for emergency services at demand node $i$ ($i \in I$). Theoretically, the probability of a server being busy should depend on the feature of the server and its competing neighboring demand nodes. Thus it is more realistic to “make use of server-specific busy fractions” [30]. However, due to the computational burden involved in the use of “server-specific busy fractions”, an intermediate approach was introduced by ReVelle and Hogan [21], that is, “use of demand-area-specific busy fractions”. The busy fraction in the service region around demand node $i$ for a particular type of emergency service vehicle (e.g. fire engine) is defined as the required service time in the region divided by the available service time in the region [30]. Thus, we have

$$q_i^E = \frac{\tilde{r}^E \sum_{j \in \text{CTI}_i} f_i}{24 \sum_{j \in \text{CTI}_i} x_j^E} = \frac{\rho_i^E}{\sum_{j \in \text{CTI}_i} x_j^E} \forall i,$$ (1)

where

- $q_i^E$ is the local busy fractions for fire engines, centered at demand node $i$;
- $\tilde{r}^E$ is the average service time of fire engines on site, in hrs/call;
- $t_{ji}$ is the travel time between station $j$ and demand node $i$;
- $f_i$ is the frequency of requests for service at demand node $i$, in calls/day;
is number of fire engines located fire station \( j \);

\( S^E \) is the service standards in terms of travel time for fire engines;

\( ME_i = \{ j \mid t_{ji} \leq S^E \} \), the set of demand nodes competing for services by fire engines located within \( S^E \) of demand node \( i \);

\( NE_i = \{ j \mid t_{ji} \leq S^E \} \), the set of fire stations located within \( S^E \) of demand node \( i \)

\( \rho^{E}_i \) is utilization ratio of fire engines at demand node \( i \), as defined by ReVelle and Snyder [31].

The local estimate of busy fraction for fire truck and ambulance can be similarly expressed by using their associated service standards.

Following the work by ReVelle and Hogan [21], we assume that the requests for services from different nodes are independent and all demand nodes within \( S^E \) have the same \( q_i^E \) value. The probability of one or more servers being busy thus follows binomial distribution. The probability of having at least one fire engine available is therefore

\[
1 - P \left[ \text{all engines within } S^E \text{ of node } i \text{ are busy} \right] = 1 - \left[ q_i^E \right]^{\sum_{j \in ME_i} x_j^E} = 1 - \left[ \frac{\rho_i^E}{\sum_{j \in NE_i} x_j^E} \right]^\sum_{j \in ME_i} x_j^E \quad \text{(2)}
\]

To meet the server availability requirement, that the probability of having at least one fire engine available within \( S^E \) of node \( i \) when node \( i \) is requesting for a service must be larger than or equal to \( \alpha \), we have

\[
1 - \left[ \frac{\rho_i^E}{\sum_{j \in NE_i} x_j^E} \right]^\sum_{j \in ME_i} x_j^E \geq \alpha \quad \text{(3)}
\]

This probabilistic constraint does not have analytical linear equivalent. However, a numerical linear equivalent can be found by defining the parameters \( e_i \) as the smallest integers which satisfy:

\[
1 - \left[ \frac{\rho_i^E}{e_i} \right]^\sum_{j \in ME_i} x_j^E \geq \alpha \quad \text{(4)}
\]

Similar expressions can be written for trucks and ambulances, substituting \( E \) by \( T \) and \( A \) respectively. Correspondingly, new parameters \( t_i \) and \( a_i \) are defined as the minimum number of fire trucks and ambulances that must be located within \( S^T \) and \( S^A \) of node \( i \), to ensure that node \( i \) is covered with reliability level \( \alpha \).

The complete mixed integer model formulation for the maximum coverage problem is

Maximize \[ \sum_{i \in D} y_i \] \( \quad \text{(5)} \)
Subject to

\begin{align}
\sum_{j \in NE_i} x_j^E & \geq e_i y_i, \forall i \in I \tag{6} \\
\sum_{j \in NT_i} x_j^T & \geq t_i y_i, \forall i \in I \tag{7} \\
\sum_{j \in NA_i} x_j^A & \geq a_i y_i, \forall i \in I \tag{8} \\
\sum_{j \in J} x_j^E & \leq P^E \tag{9} \\
\sum_{j \in J} x_j^T & \leq P^T \tag{10} \\
\sum_{j \in J} x_j^A & \leq P^A \tag{11} \\
0 & \leq x_j^E \leq B, \forall j \in J \tag{12} \\
0 & \leq x_j^A \leq B, \forall j \in J \tag{13} \\
0 & \leq x_j^T \leq B, \forall j \in J \tag{14} \\
y_i & = 0,1, \forall i \in I \tag{15}
\end{align}

where,

- $y_i = 1$ if demand node $i$ is covered by $e_i$ fire engines within $S^E$, $t_i$ fire trucks within $S^T$, and $a_i$ ambulances within $S^A$, otherwise, $y_i = 0$;
- $x_j^E$ = the number of fire engines located at station $j$;
- $x_j^A$ = the number of ambulances located at station $j$;
- $x_j^T$ = the number of fire trucks located at station $j$;
- $NE_i$, $NT_i$ and $NA_i$ = the sets of stations located within $S^E$, $S^T$ and $S^A$ of demand node $i$, respectively; e.g. $NE_i = \{ j \mid t_j \leq S^E \}$;
- $B$ = the maximum number of vehicles of each type that can be accommodated by each station. It is assumed that every candidate station has the same capacity to house $B$ fire engines, $B$ fire trucks and $B$ ambulances;
- $P^E$, $P^T$, and $P^A$ = the total number of available fire engines, fire trucks and ambulances, respectively.

The objective presented in expression (5) maximizes the total number of covered CTIs. Constraint (6) states that at each node $i$, the number of fire engines located at fire stations within $S^E$ of node $i$ must be greater than or equal to the number of fire engines needed within $S^E$ of node $i$ to meet the reliability requirement. Constraints (7-8) can be similarly explained for fire trucks and ambulances respectively. Constraints (9)-(11) restrict the total number of fire engines, fire trucks and ambulances to be assigned. Inequalities (12-14) impose the capacity constraints at each fire station.

A detailed procedure for parameter preparation in the base model is provided as following:
1. Use GIS software, e.g. ArcGIS 8.0 in our study, to locate CTI nodes and to integrate these nodes to the transportation network.

2. Generate travel time matrices containing travel times between each pair of CTI nodes and travel times between each CTI node and candidate fire station location. In this study, we wrote a script to carry out this task in ArcView 3.1.

3. Generate node sets $NE_i$, $NT_i$ and $NA_i$ for each CTI node using travel time information given in the Matrices obtained from Step 2. These sets of nodes indicate the set of candidate fire stations within the service range of CTI node $i$.

4. Generate node sets $ME_i$, $MT_i$ and $MA_i$ for each CTI node using travel time information given in the Matrices obtained from Step 2. These sets of nodes indicate the set of CTI nodes competing emergency service with CTI node $i$.

5. Compute local server busy fraction for each CTI node according to Equation 1. Once the busy fraction is computed, the minimum number of emergency service vehicles can be computed according to Inequality 4.

2.2 Robust Optimization Model

The parameters in the base model are all assumed known. However, some of the parameters such as travel time over the traffic network and the incident rate at demand nodes may be uncertain in reality. An optimal policy can be computed from a mathematical model using forecasted most likely parameters. However, the future realized values of the parameters are often different from the forecasted ones, while the effectiveness of a decision is often evaluated in the aftermath of a realized scenario as if all the parameters were known in advance. Significant data uncertainty of the decision environment and strong desire of decision makers to avoid extreme bad consequence naturally lead to consideration of robust approaches, which emphasize on the worst case scenario.

If the model parameters such as travel time over the traffic network were known in advance, one could input their values to the base model and achieve the best possible coverage. The difference between the best possible objective value and the realized objective value from a chosen strategy is called “regret” of the strategy in that realization [27]. Some robust optimization approaches deal directly with the objective values across all possible realizations. In this case, the criterion is to find a strategy that maximizes the worst benefit across all possible realizations, also called absolute robustness criterion. Some robust approaches use robust deviation criterion, which is to minimize the largest regret [28]. In this section, we will model the facility location problem using the absolute robustness criterion, i.e. we will produce an allocation strategy so as to maximize the minimum coverage for CTI nodes across all possible realizations.

Several parameters in the base model can be uncertain. Here, we will only consider uncertain travel times over the network as an illustration. The mathematical formulation is

Maximize $M$

Subject to

$$\sum_{j \in ME_i} x_{ij}^k \geq e_i y_i^k, \forall i \in I, \forall k \in K$$  \hspace{1cm} (18)

$$\sum_{j \in NT_i} x_{ij}^T \geq t_i y_i^T, \forall i \in I, \forall k \in K$$  \hspace{1cm} (19)
\[
\sum_{j \in N(A)} x_{ij}^k \geq a_i y_i^k, \forall i \in I, \forall k \in K
\] (20)

\[
\sum_{i \in I} y_i^k \geq M, \forall k \in K
\] (21)

including constraints (9)-(15)

where,

\( t_{ji}^k \) = the travel time between \( i \) and \( j \) in realization \( k \);

\( NE_i^k, NT_i^k \) and \( NA_i^k \) = the sets of stations located within \( S^E, S^T \) and \( S^A \) of demand node \( i \) in realization \( k \), respectively; e.g. \( NE_i^k = \{ j \mid t_{ji}^k \leq S^E \} \);

\( y_i^k = 1 \) if demand node \( i \) is covered by \( e_i \) fire engines within \( S^A \), \( t_i \) fire trucks within \( S^T \), and \( a_i \) ambulances within \( S^A \) under realization \( k \), otherwise, \( y_i^k = 0 \);

\( M \) = the smallest total coverage achieved in any realization;

\( K \) = set of possible realizations.

All other variables have the same meanings as in Base Model Formulation. The left side of constraint (18) represents the supply of fire engines around demand node \( i \) in scenario \( k \). The right side of constraint (18) is the demand of fire engines at demand node \( i \), which is assumed to be deterministic. Constraints (18-20) guarantee that the computed \( y_i \) is the worst coverage of demand node \( i \) across all possible scenarios. The objective of the model is still to maximize the total coverage of CTI nodes, but because of the changing meaning of \( y_i \), the objective becomes to find an allocation strategy that maximizes the minimum total coverage achieved across all scenarios. The rest of constraints are the same as constraints (9-15) in the base model formulation.

3. CASE STUDY

3.1 Background

The setting of Singapore is used as an example to illustrate the application of the proposed model to high-density metropolitan area. Singapore island has 15 fire stations and 151 CTIs. See Figure 1. The CTIs include mass rapid transit stations, transit and/or bus interchanges, bus terminals, expressway tunnels and interchanges, seaport and airport terminals.

The Singapore Civil Defense Force (SCDF) is the government agency responsible for providing emergency response services. The SCDF operates fire engines and trucks as well as a fleet of 30 ambulances. The three types of vehicles are based at fire stations. Current published service standards of SCDF are 8 minutes for fire engines and fire trucks, and 11 minutes for ambulances to reach the incident site [32]. The average service times for fire engines, fire trucks and ambulances are set to be 2, 2, and 1.5 hours respectively, which are adopted from ReVelle and Marianov [25]. A total of 3912 fire cases during the period from January 2003 to December 2003 [33] are used as historic data to estimate \( f_i \), the incident frequency at each demand node. The service reliability required by SCDF is 90%.

3.2 Base Scenario

The base scenario is based on existing practice of SCDF, in which no more than 15 fire engines, 15 fire trucks and 30 ambulances (i.e. \( p^E =15 \), \( p^T =15 \), and \( p^A =30 \)) are allocated among the 15
candidate fire stations to maximize the coverage of the 151 CTI nodes. An optimal solution to the base scenario is given in Table 2.

The existing total amount of emergency service vehicles can cover at most 126 CTI nodes if the resource is allocated optimally. An optimal allocation strategy is given in the last three columns of Table 2. Several observations are made from these results. First, the existing setting of service resources and candidate stations in Singapore is not sufficient to cover all CTI nodes under the SCDF requirements for service standard and service reliability. Second, all fire engines and trucks are fully utilized at optimal solution, but there is some redundancy with regard to the number of available ambulances. This indicates a need for redistributing the share of the three types of vehicles to achieve better system coverage. Lastly, the results indicate multiple optimal allocation strategies that lead to the same maximum coverage. Note that all the constraints and unit benefit for fire engines and trucks are the same. Thus we expect identical allocation strategies for these two types of vehicles. However, there is slight difference between the second and the third columns in Table 1 for stations 4 and 13. We later switched these two allocation strategies for the two types of vehicles and obtained the same maximum coverage of 126. Multiple optimal solutions usually provide alternatives thus more flexibility in the actual allocation of service resources.

3.3 Sensitivity Analysis in Resource Budget

As observed in the base scenario, on one hand, the total existing amount of service resources are not sufficient to cover all CTI nodes. On the other hand, some existing resources are not fully utilized. In this section, we will study the effects of total resource budget on the maximum coverage via sensitivity analysis, and also changing budget constraints to allow redistribution of the three types of resources. Usually, different types of service vehicle resources are planned by different agencies or divisions. We demonstrate that, through a more efficient budget allocation among the agencies or divisions, it is possible to achieve a higher coverage.

We combine constraints on multiple types of service vehicles (constraints 9-11) to the following single monetary resource constraint

\[
c^E \sum_{j \in J} x^E_j + c^T \sum_{j \in J} x^T_j + c^A \sum_{j \in J} x^A_j \leq I
\]

where,

- \(I\) = total amount of investment
- \(c^E, c^T,\) and \(c^A\) = unit purchasing prices of fire engine, fire truck, and ambulance, respectively.

According to SCDF, these parameters take the value of 325, 700 and 200 (in thousand US dollars), respectively.

In the base scenario, the optimal strategy requires a total of 15 fire engines, 15 fire trucks, and 21 ambulances. The corresponding cost is 19.6 million US dollars and the corresponding coverage is 126. When redistribution of resources is allowed, only 16.9 million US dollars is needed to achieve the same coverage. Furthermore, the coverage can be improved to 127 with 18.7 million dollars. The ability to cover more demand with less money shows the benefit of allowing redistribution of three types of resources.

Policy makers are often more interested in what funding level they should request instead of detail allocation strategies. Justification of certain funding level requires a cost-benefit study that can be used to measure the marginal change of coverage as total amount of funding changes. We study the range of total investments from 2.5 to 19 million US dollars with increment of 0.3
million dollars. The relationship between the total investment and the maximum coverage is illustrated in Figure 2.

In Figure 2, every data point denotes the coverage corresponding to an investment. As the total investment increases, the optimal coverage reaches a maximum of 127 and does not increase any more no matter how much investment is made. A close investigation of the spatial relationship between the potential station sites and demand sites suggests that some of the demand nodes are beyond the service areas (in terms of service standard and reliability) of the existing stations. Simply purchasing more vehicles does not improve the coverage of those demand nodes. New stations must be planned to cover those remote areas. Another clear observation from Figure 2 is that the marginal benefit of increasing one unit of investment varies across the investment levels. Therefore, when making investment decision, a balance between safety and efficiency need to be sought.

3.4 Robust Analysis
The sensitivity analysis given above is conducted in a deterministic environment, in which travel times of emergency vehicles are assumed to be known and equal to their free flow speeds. In this section, we will let the travel times between CTI nodes and fire stations fluctuate. This fluctuation may be caused by congestion, or possible unavailability of some road segments following natural or human-induced disasters. Generally, noise can be positive or negative. However, since the travel times used in the base scenario are already free flow speed, we will only consider positive noise with a uniform distribution between [0, 1], i.e., travel time between \([t, t(1+n)]\) where \(t\) is the free-flow travel time and \(n\) is the noise level expressed in percent. Three levels of noises (20%, 50% and 100%) are considered. Higher noise level implies a more congested traffic network.

For each noise level, 100 realizations with random travel times are simulated. Assume that we know in advance the actual travel times in each realization, we compute the best resource allocation strategy and the corresponding maximum coverage from the base model. The maximum coverage in each realization is plotted in Figure 3 and their statistics are given in the 2nd-4th columns in Table 3.

We then present two different strategies: expected strategy and robust strategy with their associated regrets. For each given noise level, 100 independent sets of travel time matrices were simulated. The average travel times of the 100 realizations were entered to the maximum expectation model to find the optimal expected strategy. The same sets of realized travel times were also used to compute the robust strategy. The regrets of the two strategies and their statistics are given in Table 3.

Let us consider the effects of uncertain environment on the quality of decision strategies. Whether a model is sensitive to the change of its parameter can be examined by observing the change in objective value caused by change in model parameters. In this regard, the proposed base model is not sensitive to the fluctuation in traffic network performance in the sense that the average coverage only drops by 5.9% ((121.38-114.24)/121.38) as the noise level of travel time increased from 20% to 50%. However, it is consistently observed that the performance of both strategies degrades as the level of uncertainty of the environment increases. Furthermore, as the environment uncertainty level becomes higher (noise level reaches 50% or 100%), the overall performance of robust strategy is better than the expected strategy.

Conceptually, regret can be considered as a measure of the value of perfect information, i.e., the benefit of having perfect information of uncertain model parameters. If a decision maker
prefers the base model for its conceptual simplicity, s/he should be willing to pay more for data forecasting and calibration since the benefit from better data quality is significant.

However, these observations are based on a relatively small size of sample. Much more computer simulations have to be conducted in order to draw more representative conclusion of the uncertain environment.

4. DISCUSSION
In this paper, we have studied a facility location and resource allocation problem in the special context of emergency management. Based on our dialog with practitioners in the field of emergency management, we learned that the role of location problem models is often underplayed in practice because it is considered unrealistic to relocate existing emergency service stations unless it is for new areas under developing. Via the case study, we have shown that a location model with a resource allocation feature can be used not just for identifying the best location for potential stations, but also the distribution of the service resources among the stations. Therefore, even for developed areas already equipped with service stations, it may still be valuable to reexamine whether the limited resources are distributed most efficiently. The sensitivity analysis on resource constraint also provides quantitative method for justifying the investment level for equipping emergency stations in a given region. The regret analysis in computer simulations provides value of improving prediction of uncertain model parameters and helps justify the need for improving data quality.

From modeling viewpoint, we focused mainly on handling resource availability and environment uncertainty. A base model using expected values of the model parameters and a robust model focusing on the worst-case scenario are studied at parallel. Both models are approximation of reality thus involving simplification and assumptions. The base model is simpler both conceptually and computationally. However, when significant uncertainty is involved in model parameters, following robust approach tends to be a safer choice.

In this work, we have focused on allocation of three types of emergency resources among fire stations. However, the methods presented herein are suitable for other types of emergency services and management centers such as planning shelters following large scale urban disasters and allocating inspection or medical treatment resources and personnel. Despite of the intense data processing and modeling efforts involved in this work, we have also left several import issues unaddressed. First, even though the concept of service reliability is one way to handle the uncertainty in demand, the minimum service resource requirements at demand nodes are computed based on historical data. The fluctuation of future demand due to population, unknown risk or spatial features of the study area should be considered explicitly in the model. Robust analysis of the proposed model against demand fluctuation is necessary. Secondly, the current work only provides single layer coverage. However, for highly critical infrastructures as an example, single layer coverage may not be enough to support under extreme environment. Thus, introduction of backup coverage for those infrastructures with implementation of robust optimization approaches will be an interesting extension of this work.

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<tr>
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<td>Set covering: minimize the cost of facility</td>
<td>• Specified level of coverage obtained;</td>
<td>Identify EMS vehicles locations [16-17]</td>
</tr>
<tr>
<td></td>
<td>location [14-15]:</td>
<td>• Given acceptable service distance/time</td>
<td></td>
</tr>
<tr>
<td>P-Median problem</td>
<td>Minimize the total travel distance/time between</td>
<td>• Full coverage obtained;</td>
<td>• Ambulance position for campus emergency service [19];</td>
</tr>
<tr>
<td></td>
<td>demands and facilities [18]</td>
<td>• Limited resource</td>
<td>• Locate fire stations for emergency services in Barcelona [20]</td>
</tr>
<tr>
<td>P-Center problem</td>
<td>Minimize the maximum distance between any</td>
<td>• Full coverage obtained;</td>
<td>Locate EMS vehicles with reliability requirement [21]</td>
</tr>
<tr>
<td></td>
<td>demand and its nearest facility</td>
<td>• Limited resource</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 2 Solutions of Base Scenario

<table>
<thead>
<tr>
<th>Covered nodes</th>
<th>Fire Engines (15)</th>
<th>Fire Trucks (15)</th>
<th>Ambulances (21)</th>
</tr>
</thead>
<tbody>
<tr>
<td>126</td>
<td>1, 2, 3, 6, 7, 9, 11, 12, 13, 14</td>
<td>1, 2, 3, 4, 6, 7, 9, 11, 12, 13, 14</td>
<td>1, 3, 4, 5, 6, 7, 8, 11, 12, 13</td>
</tr>
</tbody>
</table>

*Note: 1^2 means that two fire engines should be allocated at fire station 1.*
TABLE 3 Statistics of Regrets of Maximum Expected Strategy and Robust Strategy

<table>
<thead>
<tr>
<th>Noise Level</th>
<th>Maximum Coverage in Simulated Realizations</th>
<th>Regret of Expected Strategy</th>
<th>Regret of Robust Strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>(20%)</td>
<td>noise level</td>
<td>(50%)</td>
<td>noise level</td>
</tr>
<tr>
<td>Average</td>
<td>121.38</td>
<td>114.24</td>
<td>100.22</td>
</tr>
<tr>
<td>Min</td>
<td>119</td>
<td>110</td>
<td>94</td>
</tr>
<tr>
<td>Max</td>
<td>124</td>
<td>119</td>
<td>107</td>
</tr>
<tr>
<td>SD</td>
<td>1.12</td>
<td>1.77</td>
<td>2.67</td>
</tr>
<tr>
<td>Range</td>
<td>5</td>
<td>9</td>
<td>13</td>
</tr>
</tbody>
</table>
FIGURE 1  Map of Singapore with candidate fire stations
FIGURE 2 Sensitivity analysis on total investment
FIGURE 3 Best possible coverage in each realization