What You Should Learn

• Plot points in the Cartesian plane.

• Use the Distance Formula to find the distance between two points.

• Use the Midpoint Formula to find the midpoint of a line segment.

• Use a coordinate plane to model and solve real-life problems.
The Cartesian Plane
The Cartesian Plane

Just as you can represent real numbers by points on a real number line, you can represent ordered pairs of real numbers by points in a plane called the **rectangular coordinate system**, or the **Cartesian plane**, named after the French mathematician René Descartes.

The Cartesian plane is formed by using two real number lines intersecting at right angles, as shown in Figure 1.1.
The horizontal real number line is usually called the \textit{x-axis}, and the vertical real number line is usually called the \textit{y-axis}.

The point of intersection of these two axes is the \textit{origin}, and the two axes divide the plane into four parts called \textit{quadrants}.

Each point in the plane corresponds to an \textit{ordered pair} \((x, y)\) of real numbers \(x\) and \(y\), called \textit{coordinates} of the point.
The \textbf{x-coordinate} represents the directed distance from the \textit{y}-axis to the point, and the \textbf{y-coordinate} represents the directed distance from the \textit{x}-axis to the point, as shown in Figure 1.2.

The notation \((x, y)\) denotes both a point in the plane and an open interval on the real number line. The context will tell you which meaning is intended.
Example 1 – *Plotting Points in the Cartesian Plane*

Plot the points \((-1, 2), (3, 4), (0, 0), (3, 0),\) and \((-2, -3)\).

**Solution:**

To plot the point \((-1, 2)\), imagine a vertical line through \(-1\) on the *x*-axis and a horizontal line through 2 on the *y*-axis.

The intersection of these two lines is the point \((-1, 2)\).

The other four points can be plotted in a similar way, as shown in Figure 1.3.
The beauty of a rectangular coordinate system is that it allows you to see relationships between two variables.

It would be difficult to overestimate the importance of Descartes’s introduction of coordinates in the plane.
The Pythagorean Theorem and the Distance Formula
The following famous theorem is used extensively throughout this course.

**Pythagorean Theorem**
For a right triangle with hypotenuse of length \( c \) and sides of lengths \( a \) and \( b \), you have \( a^2 + b^2 = c^2 \), as shown in Figure 1.5. (The converse is also true. That is, if \( a^2 + b^2 = c^2 \), then the triangle is a right triangle.)

![Figure 1.5](image-url)
The Pythagorean Theorem and the Distance Formula

Suppose you want to determine the distance $d$ between two points $(x_1, y_1)$ and $(x_2, y_2)$ in the plane. With these two points, a right triangle can be formed, as shown in Figure 1.6.

![Figure 1.6](image-url)
The Pythagorean Theorem and the Distance Formula

The length of the vertical side of the triangle is $|y_2 - y_1|$, and the length of the horizontal side is $|x_2 - x_1|$.

By the Pythagorean Theorem, you can write

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$ 

This result is the **Distance Formula**.

---

**The Distance Formula**

The distance $d$ between the points $(x_1, y_1)$ and $(x_2, y_2)$ in the plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$
Example 3 – *Finding a Distance*

Find the distance between the points \((-2, 1)\) and \((3, 4)\).

**Solution:**

Let \((x_1, y_1) = (-2, 1)\) and \((x_2, y_2) = (3, 4)\). Then apply the Distance Formula.

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

Distance Formula

\[
= \sqrt{[3 - (-2)]^2 + (4 - 1)^2}
\]

Substitute for \(x_1, y_1, x_2,\) and \(y_2\).

\[
= \sqrt{(5)^2 + (3)^2}
\]

Simplify.
Example 3 – Solution

So, the distance between the points is about 5.83 units.

You can use the Pythagorean Theorem to check that the distance is correct.

\[
\begin{align*}
(\sqrt{34})^2 & \equiv 3^2 + 5^2 \\
34 & \equiv 34
\end{align*}
\]

Pythagorean Theorem

Substitute for \(d\).

Distance checks. ✓
The Midpoint Formula
To find the **midpoint** of the line segment that joins two points in a coordinate plane, you can simply find the average values of the respective coordinates of the two endpoints using the **Midpoint Formula**.

**The Midpoint Formula**

The midpoint of the line segment joining the points \((x_1, y_1)\) and \((x_2, y_2)\) is given by the Midpoint Formula

\[
\text{Midpoint} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]
Example 5 – Finding a Line Segment’s Midpoint

Find the midpoint of the line segment joining the points (−5, −3) and (9, 3).

Solution:
Let \((x_1, y_1) = (−5, −3)\) and \((x_2, y_2) = (9, 3)\).

Midpoint = \(\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)\)

= \(\left( \frac{−5 + 9}{2}, \frac{−3 + 3}{2} \right)\)

Midpoint Formula
Substitute for \(x_1, y_1, x_2, \) and \(y_2\).
Example 5 – Solution

\[
\begin{align*}
\text{Simplify.} \\
= (2, 0)
\end{align*}
\]

The midpoint of the line segment is \((2, 0)\), as shown in Figure 1.9.
Applications
Example 6 – Finding the Length of a Pass

A football quarterback throws a pass from the 28-yard line, 40 yards from the sideline. The pass is caught by a wide receiver on the 5-yard line, 20 yards from the same sideline, as shown in Figure 1.10. How long is the pass?
Example 6 – Solution

You can find the length of the pass by finding the distance between the points (40, 28) and (20, 5).

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

Distance Formula

\[ = \sqrt{(40 - 20)^2 + (28 - 5)^2} \]

Substitute for \( x_1, y_1, x_2, \) and \( y_2. \)

\[ = \sqrt{400 + 529} \]

Simplify.

\[ = \sqrt{929} \]

Simplify.

\[ \approx 30 \]

Use a calculator.

So, the pass is about 30 yards long.
In Example 6, the scale along the goal line does not normally appear on a football field.

However, when you use coordinate geometry to solve real-life problems, you are free to place the coordinate system in any way that is convenient for the solution of the problem.