1.6 A LIBRARY OF PARENT FUNCTIONS
What You Should Learn

• Identify and graph linear and squaring functions.

• Identify and graph cubic, square root, and reciprocal function.

• Identify and graph step and other piecewise-defined functions.

• Recognize graphs of parent functions.
Linear and Squaring Functions
The graph of the **linear function** \( f(x) = ax + b \) is a line with slope \( m = a \) and \( y \)-intercept at \((0, b)\).
Characteristics of the Linear Function:

- Domain: the set of all real numbers.
- Range: the set of all real numbers.
- x-intercept: \((-\frac{b}{m}, 0)\)
- y-intercept: \((0, b)\).
- The graph is increasing if \(m > 0\), decreasing if \(m < 0\), and constant if \(m = 0\).
Example 1 – Writing a Linear Function

Write the linear function for which \( f(1) = 3 \) and \( f(4) = 0 \).

Solution:
To find the equation of the line that passes through \((x_1, y_1) = (1, 3)\) and \((x_2, y_2) = (4, 0)\) first find the slope of the line.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{4 - 1} = \frac{-3}{3} = -1
\]
Next, use the point-slope form of the equation of a line.

\[ y - y_1 = m(x - x_1) \]  \hspace{1cm} \text{Point-slope form}

\[ y - 3 = -1(x - 1) \]  \hspace{1cm} \text{Substitute for } x_1, y_1 \text{ and } m

\[ y = -x + 4 \]  \hspace{1cm} \text{Simplify.}

\[ f(x) = -x + 4 \]  \hspace{1cm} \text{Function notation}
Example 1 – Solution

The graph of this function is shown in Figure 1.65.

\[ f(x) = -x + 4 \]
Constant Function

\[ f(x) = c \]

Domain: all real numbers
Range: a single real number \( c \).
Graph: a horizontal line
Identity Function

\[ f(x) = x \]

Domain: the set of all real numbers
Range: the set of all real numbers
Graph: a line with \( m = 1 \) and a \( y \)-intercept at \((0, 0)\).

a line for which each \( x \)-coordinate equals the corresponding \( y \)-coordinate.
increasing
Squaring Function

\[ f(x) = x^2 \]

**Graph:** U-shaped curve with the following characteristics.

- The domain of the function is the set of all real numbers.
- The range of the function is the set of all nonnegative real numbers.
- The function is even.
Linear and Squaring Functions

• The graph has an intercept at (0, 0).

• The graph is decreasing on the interval \((-\infty, 0)\) and increasing on the interval \((0, \infty)\).

• The graph is symmetric with respect to the y-axis.

• The graph has a relative minimum at (0, 0).
Cubic, Square Root, and Reciprocal Functions
Cubic, Square Root, and Reciprocal Functions

1. The graph of the **cubic function** \( f(x) = x^3 \) has the following characteristics.

   • Domain: the set of all real numbers.
   • Range: the set of all real numbers.
   • The function is odd.
   • The graph has an intercept at \((0, 0)\).
Cubic, Square Root, and Reciprocal Functions

- The graph is increasing on the interval \((-\infty, \infty)\).
- The graph is symmetric with respect to the origin.
2. The graph of the **square root** function \( f(x) = \sqrt{x} \) has the following characteristics.

- **Domain**: the set of all nonnegative real numbers.
- **Range**: the set of all nonnegative real numbers.
- The graph has an intercept at \((0, 0)\).
- The graph is increasing on the interval \((0, \infty)\).
3. The graph of the **reciprocal function** following characteristics.

- Domain: \((-\infty, 0) \cup (0, \infty)\)
- Range: \((-\infty, 0) \cup (0, \infty)\)
- Odd.
- No intercepts.
- The graph is decreasing on the intervals \((-\infty, 0)\) and \((0, \infty)\).
- The graph is symmetric with respect to the origin.
Step and Piecewise-Defined Functions
Step and Piecewise-Defined Functions

Functions whose graphs resemble sets of stairsteps are known as **step functions**.

**greatest integer function,**

\[ f(x) = \left\lfloor x \right\rfloor = \text{the greatest integer less than or equal to } x. \]
Step and Piecewise-Defined Functions

\[ \lceil -1 \rceil = (\text{greatest integer } \leq -1) = -1 \]

\[ \lceil -\frac{1}{2} \rceil = (\text{greatest integer } \leq -\frac{1}{2}) = -1 \]

\[ \lceil \frac{1}{10} \rceil = (\text{greatest integer } \leq \frac{1}{10}) = 0 \]

\[ \lceil 1.5 \rceil = (\text{greatest integer } \leq 1.5) = 1 \]
Step and Piecewise-Defined Functions

\[ f(x) = \lfloor x \rfloor \]

• Domain: the set of all real numbers.
• Range of: the set of all integers.
• \( y \)-intercept: (0, 0)
• \( x \)-intercepts: interval \([0, 1)\).
Example 2 – Evaluating a Step Function

Evaluate the function when \( x = -1, 2 \) and \( \frac{3}{2} \).

\[ f(x) = \lfloor x \rfloor + 1 \]

Solution:

For \( x = -1 \), the greatest integer \( \leq -1 \) is \(-1\), so

\[ f(-1) = \lfloor -1 \rfloor + 1 \]

\[ = -1 + 1 \]

\[ = 0 \]
Example 2 – Solution

For $x = 2$, the greatest integer $\leq 2$ is 2, so

\[ f(2) = \left\lfloor 2 \right\rfloor + 1 \]

\[ = 2 + 1 \]

\[ = 3. \]

For $x = \frac{3}{2}$, the greatest integer $\leq \frac{3}{2}$ is 1, so

\[ f\left(\frac{3}{2}\right) = \left\lfloor \frac{3}{2} \right\rfloor + 1 \]

\[ = 1 + 1 \]

\[ = 2 \]
Example 2 – Solution

You can verify your answers by examining the graph of $f(x) = \lfloor x \rfloor + 1$ shown in Figure 1.73.
Parent Functions
The eight graphs shown in Figure 1.75 represent the most commonly used functions in algebra.

- **(a) Constant Function**
  \[ f(x) = c \]

- **(b) Identity Function**
  \[ f(x) = x \]

- **(c) Absolute Value Function**
  \[ f(x) = |x| \]

- **(d) Square Root Function**
  \[ f(x) = \sqrt{x} \]
Parent Functions

(e) Quadratic Function

\( f(x) = x^2 \)

(f) Cubic Function

\( f(x) = x^3 \)

(g) Reciprocal Function

\( f(x) = \frac{1}{x} \)

(h) Greatest Integer Function

\( f(x) = [x] \)