1.7 TRANSFORMATIONS OF FUNCTIONS

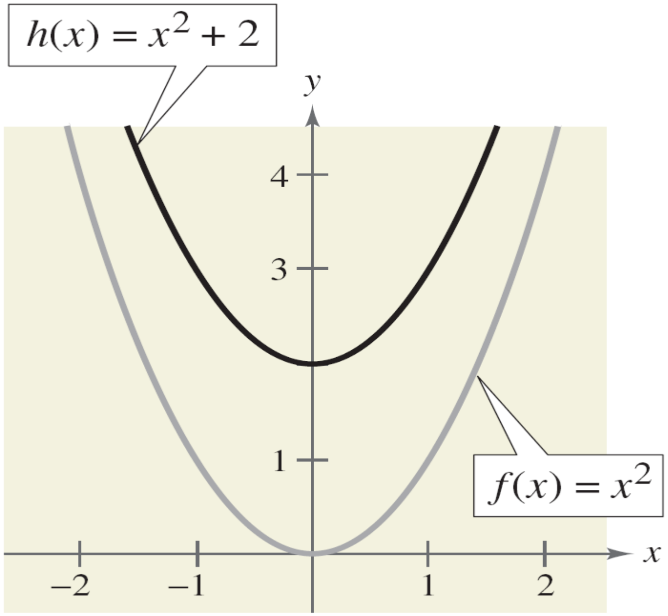
What You Should Learn

- Use vertical and horizontal shifts to sketch graphs of functions.
- Use reflections to sketch graphs of functions.
- Use nonrigid transformations to sketch graphs of functions.

Shifting Graphs

Shifting Graphs

$h(x) = x^2 + 2$



Start with the graph of $f(x) = x^2$

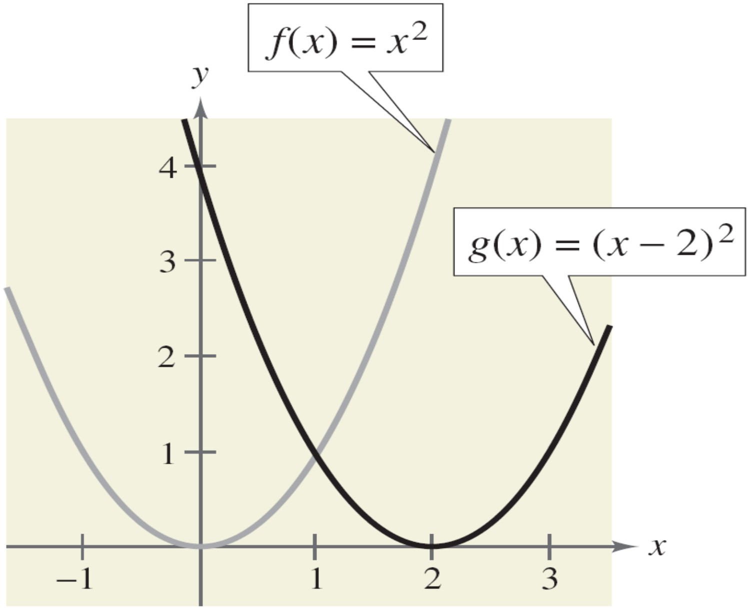
Shifting upward two units

$h(x) = x^2 + 2 = f(x) + 2$

Upward shift of two units

Shifting Graphs

$g(x) = (x - 2)^2$



Start with the graph of $f(x) = x^2$

Shifting to the *right* two units

$g(x) = (x - 2)^2 = f(x - 2)$ Right shift of two units

Figure 1.77

Shifting Graphs

Vertical and Horizontal Shifts

Let c be a positive real number. **Vertical and horizontal shifts** in the graph of $y = f(x)$ are represented as follows.

1. Vertical shift c units *upward:* $h(x) = f(x) + c$
2. Vertical shift c units *downward:* $h(x) = f(x) - c$
3. Horizontal shift c units to the *right:* $h(x) = f(x - c)$
4. Horizontal shift c units to the *left:* $h(x) = f(x + c)$

Example 1 – *Shifts in the Graphs of a Function*

Use the graph of $f(x) = x^3$ to sketch the graph of each function.

a. $g(x) = x^3 - 1$ **b.** $h(x) = (x + 2)^3 + 1$

Solution:

a. Relative to the graph of $f(x) = x^3$, the graph of

$$g(x) = x^3 - 1$$

is a downward shift of one unit, as shown in Figure 1.78.

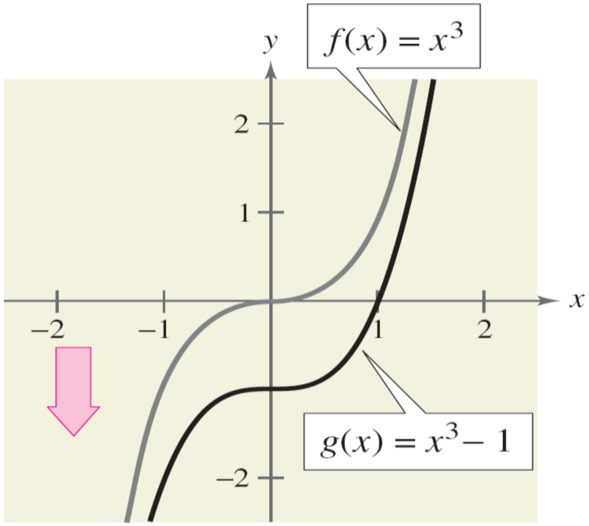


Figure 1.78

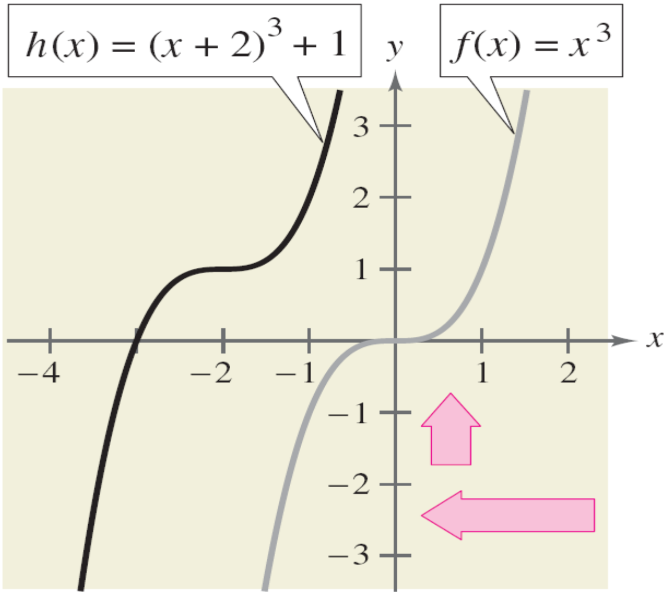
Example 1 – *Solution*

cont'd

b. Relative to the graph of $f(x) = x^3$, the graph of

$$h(x) = (x + 2)^3 + 1$$

involves a left shift of two units and an upward shift of one unit, as shown in figure .



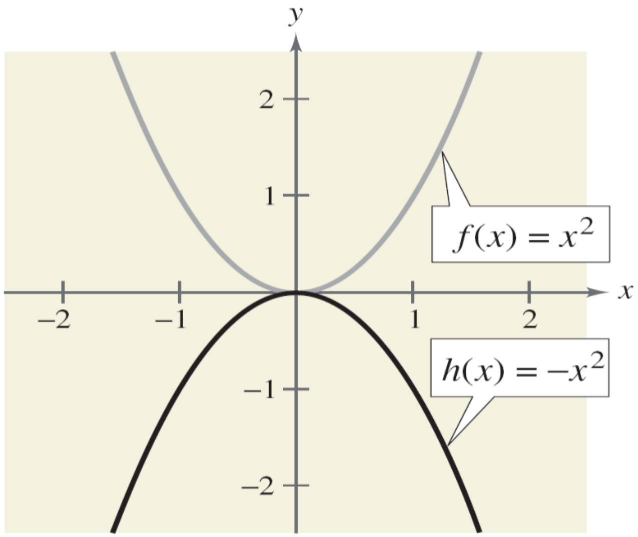
Reflecting Graphs

Reflecting Graphs

$$h(x) = -x^2$$

is the mirror image (or reflection) of the graph of

$$f(x) = x^2,$$



Reflecting Graphs

Reflections in the Coordinate Axes

Reflections in the coordinate axes of the graph of $y = f(x)$ are represented as follows.

1. Reflection in the x-axis: $h(x) = -f(x)$

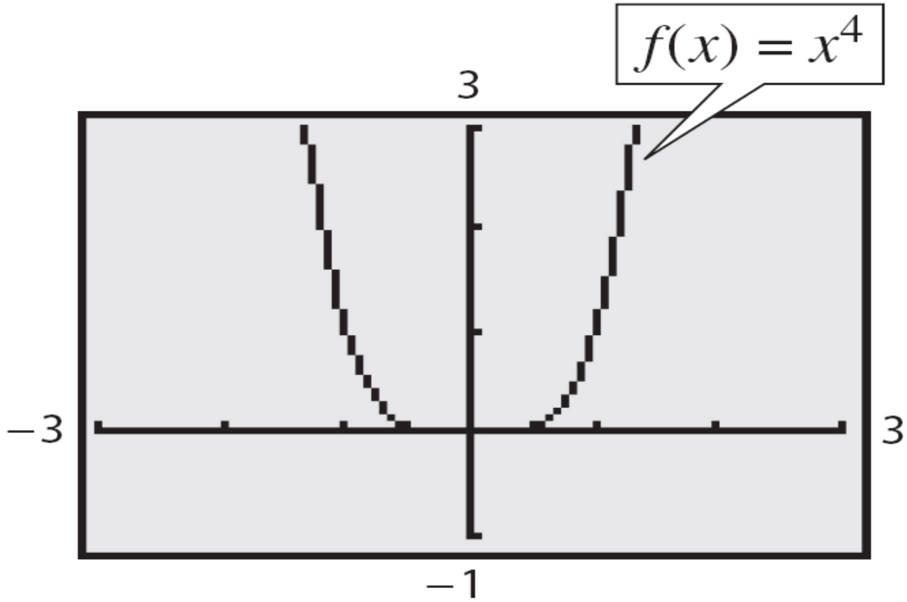
2. Reflection in the y-axis: $h(x) = f(-x)$

Example 2 – *Finding Equations from Graphs*

The graph of the function given by

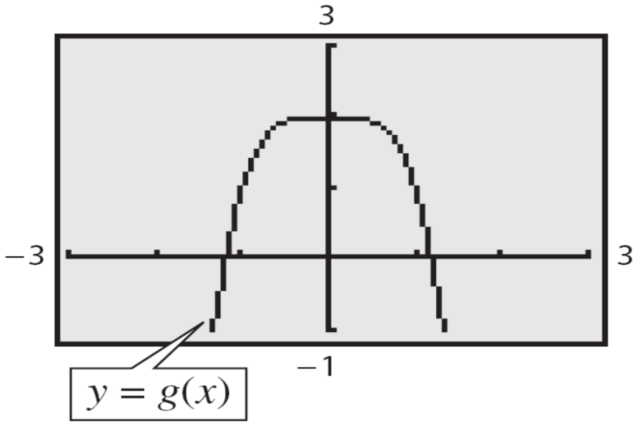
$$f(x) = x^4$$

is

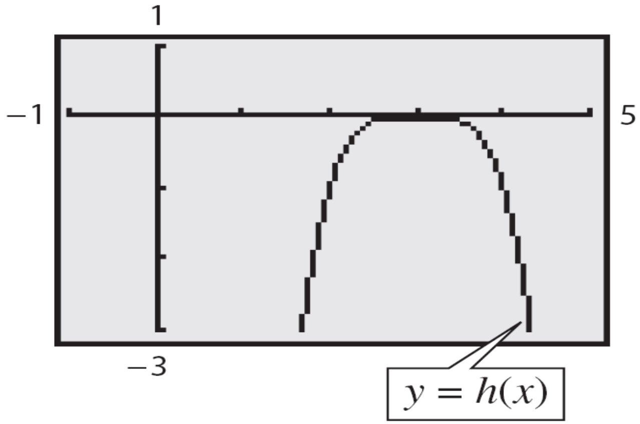


Example 2 – *Finding Equations from Graphs* cont’d

Each of the graphs in figure is a transformation of the graph of *f*. Find an equation for each of these functions.



(a)



(b)

Example 2 – *Solution*

a. The graph of g is a reflection in the x-axis *followed by* an upward shift of two units of the graph of $f(x) = x^4$.

So, the equation for is g is

$$g(x) = -x^4 + 2.$$

b. The graph of h is a horizontal shift of three units to the right *followed by* a reflection in the x-axis of the graph of $f(x) = x^4$.

So, the equation for h is

$$h(x) = -(x - 3)^4.$$

Reflecting Graphs

Function involving **square roots**, restrict the domain to **EXCLUDE** negative numbers inside the radical.

$$\text{Domain of } g(x) = -\sqrt{x}: \quad x \geq 0$$

$$\text{Domain of } h(x) = \sqrt{-x}: \quad x \leq 0$$

$$\text{Domain of } k(x) = -\sqrt{x+2}: \quad x \geq -2$$

Nonrigid Transformations

Nonrigid Transformations

Rigid transformations: the basic shape of the graph is **unchanged**.

Horizontal shifts,

Vertical shifts,

Reflections

Change only the *position* of the graph in the coordinate plane.

Nonrigid transformations: cause a *distortion*—a change in the shape of the original graph.

Nonrigid Transformations

A nonrigid transformation of the graph of $y = f(x)$ is represented by

$$g(x) = cf(x),$$

where the transformation is a **vertical stretch** if $c > 1$ and a **vertical shrink** if $0 < c < 1$.

A nonrigid transformation of the graph of $y = f(x)$ is represented by

$$h(x) = f(cx),$$

where the transformation is a **horizontal shrink** if $c > 1$ and a **horizontal stretch** if $0 < c < 1$.

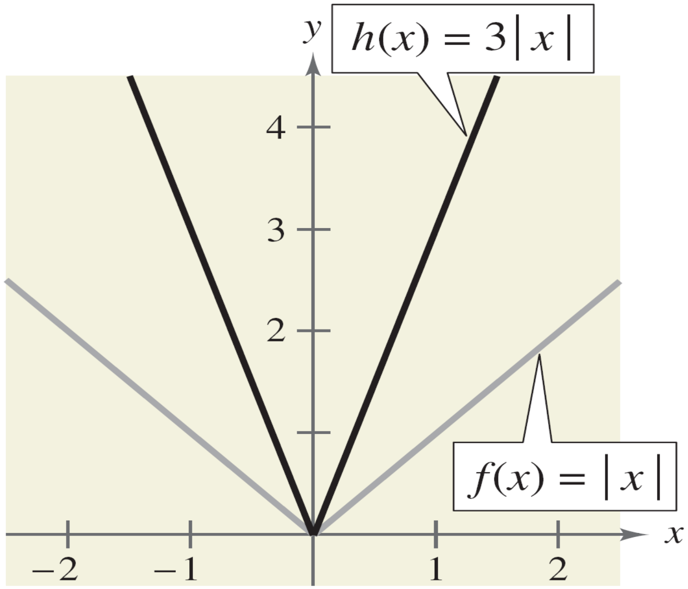
Example 4 – *Nonrigid Transformations*

Compare the graph of each function with the graph of $f(x) = |x|$.

a. $h(x) = 3|x|$ **b.** $g(x) = \frac{1}{3}|x|$

Solution:

a. Relative to the graph of $f(x) = |x|$, the graph of $h(x) = 3|x| = 3f(x)$ is a vertical stretch (each y-value is multiplied by 3) of the graph of f.



Example 4 – *Solution*

cont'd

b. Similarly, the graph of

$$g(x) = \frac{1}{3}|x| = \frac{1}{3}f(x)$$

is a vertical shrink (each y-value is multiplied by $\frac{1}{3}$) of the graph of f.