4.1 Radian and Degree Measure
What You Should Learn

• Describe angles.

• Use radian measure.

• Use degree measure.

• Use angles to model and solve real-life problems.
Angles
**Angles**

**Trigonometry** means “measurement of triangles.”

An **angle** is determined by rotating a ray (half-line) about its endpoint.
Angles

standard position

Positive angle (counterclockwise)
Negative angle (clockwise)

Same initial and terminal sides. Such angles are coterminated.
Radian Measure
Radian Measure

The **measure of an angle** is determined by the amount of rotation from the initial side to the terminal side. One way to measure angles is in *radians*.

To define a radian, you can use a **central angle** of a circle, one whose vertex is the center of the circle.

Arc length = radius when $\theta = 1$ radian
Radian Measure

Definition of Radian

One **radian** is the measure of a central angle $\theta$ that intercepts an arc $s$ equal in length to the radius $r$ of the circle. See Figure 4.5. Algebraically, this means that

$$\theta = \frac{s}{r}$$

where $\theta$ is measured in radians.
Radian Measure

\[
\frac{1}{2} \text{ revolution} = \frac{2\pi}{2} = \pi \text{ radians}
\]

\[
\frac{1}{4} \text{ revolution} = \frac{2\pi}{4} = \frac{\pi}{2} \text{ radians}
\]

\[
\frac{1}{6} \text{ revolution} = \frac{2\pi}{6} = \frac{\pi}{3} \text{ radians}
\]
### Radian Measure

The radian measure of an angle is the ratio of the length of the arc it subtends on a circle to the radius of the circle. It is a unit of measurement for angles, where one radian is the angle subtended at the center of a circle by an arc that is equal in length to the radius of the circle.

<table>
<thead>
<tr>
<th>Quadrant II</th>
<th>Quadrant I</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\pi}{2} &lt; \theta &lt; \pi$</td>
<td>$0 &lt; \theta &lt; \frac{\pi}{2}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quadrant III</th>
<th>Quadrant IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi &lt; \theta &lt; \frac{3\pi}{2}$</td>
<td>$\frac{3\pi}{2} &lt; \theta &lt; 2\pi$</td>
</tr>
</tbody>
</table>

- $\theta = \frac{\pi}{2}$
- $\theta = \pi$
- $\theta = \frac{3\pi}{2}$
- $\theta = 0$
A given angle $\theta$ has infinitely many coterminal angles. For instance, $\theta = \pi/6$ is coterminal with

$$\frac{\pi}{6} + 2n\pi$$

where $n$ is an integer.
Example 1 – Sketching and Finding Coterminal Angles

a. For the positive angle $13\pi/6$, subtract $2\pi$ to obtain a coterminal angle

$$\frac{13\pi}{6} - 2\pi = \frac{\pi}{6}.$$
Example 1 – Sketching and Finding Coterminal Angles

b. For the positive angle \( \frac{3\pi}{4} \), subtract \( 2\pi \) to obtain a coterminal angle

\[
\frac{\frac{3\pi}{4} - 2\pi}{4} = -\frac{5\pi}{4}.
\]
Example 1 – *Sketching and Finding Coterminal Angles*

**c.** For the negative angle \(-\frac{2\pi}{3}\), add \(2\pi\) to obtain a coterminal angle

\[-\frac{2\pi}{3} + 2\pi = \frac{4\pi}{3}\].

\[
\begin{align*}
\frac{\pi}{2} & \quad \frac{\pi}{2} \\
\frac{4\pi}{3} & \quad \pi \\
\frac{3\pi}{2} & \quad 0
\end{align*}
\]
Radian Measure

Two positive angles $\alpha$ and $\beta$ are **complementary** (complements of each other) if their sum is $\pi/2$.

Two positive angles are **supplementary** (supplements of each other) if their sum is $\pi$. 

![Complementary angles](image1)

![Supplementary angles](image2)
Degree Measure
Degree Measure

A second way to measure angles is in terms of degrees, denoted by the symbol °.

A measure of one degree (1°) is equivalent to a rotation of $\frac{1}{360}$ of a complete revolution about the vertex.
Degree Measure

When no units of angle measure are specified, **radian measure is implied**.

\[ 1^\circ = \frac{\pi}{180} \text{ rad} \quad \text{and} \quad 1 \text{ rad} = \left( \frac{180^\circ}{\pi} \right) \]
Conversions Between Degrees and Radians

1. To convert degrees to radians, multiply degrees by $\frac{\pi \text{ rad}}{180^\circ}$.

2. To convert radians to degrees, multiply radians by $\frac{180^\circ}{\pi \text{ rad}}$.

To apply these two conversion rules, use the basic relationship $\pi \text{ rad} = 180^\circ$. (See Figure 4.14.)
Example 3 – Converting from Degrees to Radians

a. \[ 135^\circ = (135 \text{ deg}) \left( \frac{\pi \text{ rad}}{180 \text{ deg}} \right) \]
   \[ = \frac{3\pi}{4} \text{ radians} \]

b. \[ 540^\circ = (540 \text{ deg}) \left( \frac{\pi \text{ rad}}{180 \text{ deg}} \right) \]
   \[ = 3\pi \text{ radians} \]

c. \[ -270^\circ = (-270 \text{ deg}) \left( \frac{\pi \text{ rad}}{180 \text{ deg}} \right) \]
   \[ = -\frac{3\pi}{2} \text{ radians} \]
Applications
Applications

Arc Length

For a circle of radius $r$, a central angle $\theta$ intercepts an arc of length $s$ given by

$$s = r\theta$$

Length of circular arc

where $\theta$ is measured in radians. Note that if $r = 1$, then $s = \theta$, and the radian measure of $\theta$ equals the arc length.
Example 5 – Finding Arc Length

A circle has a radius of 4 inches. Find the length of the arc intercepted by a central angle of $240^\circ$, as shown in Figure 4.15.
Example 5 – Solution

Convert $240^\circ$ to radian measure:

$$240^\circ = (240 \text{ deg}) \left( \frac{\pi \text{ rad}}{180 \text{ deg}} \right)$$

$$= \frac{4\pi}{3} \text{ radians}$$

Find the arc length to be

$$s = r\theta$$

$$= 4 \left( \frac{4\pi}{3} \right)$$

$$= \frac{16\pi}{3} \approx 16.76 \text{ inches.}$$
Linear and Angular Speeds

Consider a particle moving at a constant speed along a circular arc of radius $r$. If $s$ is the length of the arc traveled in time $t$, then the linear speed $v$ of the particle is

$$\text{Linear speed } v = \frac{\text{arc length}}{\text{time}} = \frac{s}{t}.$$ 

Moreover, if $\theta$ is the angle (in radian measure) corresponding to the arc length $s$, then the angular speed $\omega$ (the lowercase Greek letter omega) of the particle is

$$\text{Angular speed } \omega = \frac{\text{central angle}}{\text{time}} = \frac{\theta}{t}.$$
A sector of a circle is the region bounded by two radii of the circle and their intercepted arc.

**Area of a Sector of a Circle**

For a circle of radius $r$, the area $A$ of a sector of the circle with central angle $\theta$ is given by

$$A = \frac{1}{2}r^2\theta$$

where $\theta$ is measured in radians.