Problem: A signal $x_p(t)$ is obtained through impulse-train sampling of a sinusoidal signal $x(t)$ whose frequency is equal to half the sampling frequency $\omega_s$.

$$x(t) = \cos\left(\frac{\omega_s}{2}t + \phi\right)$$

and

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

where $T = \frac{2\pi}{\omega_s}$.

(a) Find $g(t)$ such that

$$x(t) = \cos(\phi) \cos\left(\frac{\omega_s}{2}t\right) + g(t)$$

(b) Show that

$$g(nT) = 0 \quad \text{for} \quad n = 0, \pm 1, \pm 2, \cdots$$

(c) Using the results of the previous two parts, show that if $x_p(t)$ is applied as the input to an ideal lowpass filter with cut-off frequency $\omega_s/2$, the resulting output is

$$y(t) = \cos(\phi) \cos\left(\frac{\omega_s}{2}t\right)$$